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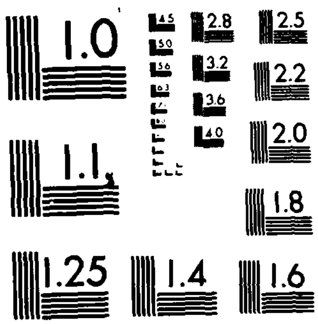

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SHOCK WAVES IN THREE-DIMENSIONAL
ELASTIC SOLIDS

FINAL REPORT

by

T. C. T. Ting

January 9, 1984

U.S. Army Research Office

DAAG 29-80-C-0093

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The research project supported by the Army Research Office was to study some aspects of shock waves in three-dimensional elastic solids. In particular, the transport equations are to be derived for the growth or decay of a three-dimensional shock wave. This report contains the summary of the results obtained from this investigation.

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I. STATEMENT OF THE PROBLEM STUDIED

The research project supported by the Army Research Office was to study some aspects of shock waves in three-dimensional elastic solids. In particular, the transport equations are to be derived for the growth or decay of a three-dimensional shock wave. This report contains the summary of the results obtained from this investigation.

II. SUMMARY OF THE RESULTS OBTAINED

Transport equations for the growth or decay of the shock amplitude in three-dimensional solids are much more difficult to derive than that for the one-dimensional solids. As a starting point, we considered shock waves in three-dimensional elastic fluids because there is only one wave speed in elastic fluids while in solids there are three wave speeds. Thus the problem is simpler than three-dimensional solids but more complicated than one-dimensional solids. Although the problem has been studied by Chen and Wright, our purpose was to see if, by a different approach, the analysis for the three-dimensional fluids can be extended to three-dimensional solids. The results obtained (see [1]) were promising. It is shown that, with the exception of the term which contains the mean curvature of the shock surface, the transport equations are almost identical to the transport equations for one-dimensional nonlinear elastic solids if we replace the stress, strain and velocity in the latter by the pressure, specific volume and normal velocity, respectively. Therefore, the results obtained for one-dimensional shock waves regarding whether the amplitudes of jump in stress, strain and velocity grow or decay simultaneously can be applied here to the jump in pressure, specific volume and normal velocity. New compatibility equations are obtained for the velocity gradients behind the shock wave. We also obtain a universal relation between the variations of amplitudes of jump in pressure, specific volume and normal velocity.

With the new approach introduced in [1], we were ready to tackle the three-dimensional solids. In studying the system of differential equations for wave propagation in general three-dimensional solids, we inadvertently discovered a very simple characteristic form of differential equations in which the stresses and velocities appeared explicitly. This is presented in [2]. The *conventional* forms of characteristic equations contain the stresses and velocities implicitly. Both Lagrangian coordinates and Eulerian coordinates are considered. The constitutive equations considered in [2] apply to a large class of nonlinear materials. The characteristic forms derived in [2] clearly show which components of the stress and velocity are involved in the differentiation along the bicharacteristics. Moreover, the reduction to one-dimensional cases from three-dimensional problems is obvious for the characteristic forms obtained here. Examples are given and compared with the known solution in the literature for wave propagation in linear isotropic elastic solids and isentropic compressible fluids.

The transport equations for the growth or decay of a shock wave in three-dimensional elastic solids were derived and presented in [3,4]. Lagrangian coordinates were used in [3] while Eulerian coordinates were used in [4]. It is shown that the growth or decay of a discontinuity ψ depends on (i) an unknown quantity ϕ^- behind the shock wave, (ii) the two principal curvatures of the shock surface, (iii) the gradient on the shock surface of the shock wave speeds and (iv) the inhomogeneous term which depends on the motion ahead of the shock surface and vanishes when the motion ahead of the shock surface is uniform. If a proper choice is made of the propagation vector $\underset{\sim}{b}$ along which the growth or decay of the discontinuity is measured, the dependence on item (iii) can be avoided. However, $\underset{\sim}{b}$ assumes different directions depending on the choice of discontinuity ψ with which one is concerned and the unknown quan-

tity ϕ^- behind the shock wave on which one chooses to depend. As in the case of one-dimensional shock waves, the growth (or decay) of one discontinuity may not be accompanied by the growth (or decay) of other discontinuities. A universal equation relating the growth or decay of discontinuities in the normal stress, normal velocity and specific volume is also presented.

Application of the transport equations to a three-dimensional initial and boundary value problem would be a very complex and difficult undertaking. In view of limited time left for the project, we considered two simple problems which are related to shock waves in nonlinear elastic solids.

The first problem is the propagation of plane waves in an elastic half space [5]. Using stress as the dependent variable instead of the deformation gradient, plane waves of finite amplitude in simple elastic solids are studied. For isotropic materials there are two plane polarized simple waves as well as shock waves and one circularly polarized simple wave which can also be regarded as a shock wave. With the aid of the stress paths for simple waves and shock waves in the stress space introduced here, one can see clearly what combination of simple waves and/or shock waves is needed to satisfy the initial and boundary conditions. We use second order isotropic hyperelastic materials to illustrate the ideas. In one example we show that the solution requires as many as four simple waves. In another we show that depending on the boundary condition there are more than eight possible solutions to the problem. We also present an example in which the solution does not depend continuously on the boundary condition. This implies that in experiments if the applied load at the boundary is not properly controlled, any slight deviation in the applied load would result in a finite different response in the material.

The second problem is the reflection of an oblique plane shock wave from a boundary in a two-dimensional isotropic hyperelastic material [6]. For plane

strain deformations, the strain energy function W is a function of two invariants p and q of the deformation gradient. There are, in general, two reflected waves each of which can be a simple wave or a shock wave. For special class of materials for which the strain energy function W(p,q) represents a developable surface (of which harmonic materials are particular examples), one of the reflected waves is always a shock wave. It is shown that there are materials other than harmonic materials for which the wave speeds are independent of the direction of propagation. Illustrative examples are presented to show how one can determine the reflected waves from a rigid boundary. It is also shown that for certain incident shock waves, there exists only one reflected wave.

It is clear that there are many important problems remained to be investigated. Nevertheless, the preliminary results obtained for this project would be useful in solving shock wave propagation in three-dimensional solids.

III. PUBLICATIONS UNDER THIS PROJECT

[1] T. C. T. Ting, 'Intrinsic Description of Three-Dimensional Shock Waves in Elastic Fluids,' Int. J. Eng. Science, Vol. 19, No. 5, 1981, 629-638.

[2] T. C. T. Ting, 'Characteristic Forms of Differential Equations for Wave Propagation in Nonlinear Media,' J. Appl. Mech., Vol. 48, No. 4, 1981, 743-748.

[3] Yongchi Li and T. C. T. Ting, 'Lagrangian Description of Transport Equations for Shock Waves in Three-Dimensional Elastic Solids,' Appl. Math. and Mech., Vol. 3, No. 4, 1982, 491-506.

[4] T. C. T. Ting and Yongchi Li, 'Eulerian Formulation of Transport Equations for Three-Dimensional Shock Waves in Simple Elastic Solids,' J. Elasticity, Vol. 13, No. 3, 1983, 295-310.

[5] Yongchi Li and T. C. T. Ting, 'Plane Waves in Simple Elastic Solids and Discontinuous Dependence of Solution on Boundary Conditions,' Int. J. Solids Structures, Vol. 19, No. 11, 1983, 989-1008.

[6] Yongchi Li and T. C. T. Ting, 'Simple Waves and Shock Waves Generated by an Incident Shock Wave in Two-Dimensional Hyperelastic Materials,' To appear in J. Appl. Mech.

IV. SCIENTIFIC PERSONNEL PARTICIPATED IN THE PROJECT

A. Graduate Students

1. R. Zwiers - received M.Sc. in December 1980
2. P. H. Hoang - received M.Sc. in August 1981
3. Zhijing Tang - received M.Sc. in June 1983.

B. Research Associate

1. Yongchi Li - March 1981 to May 1983